Explicit and Semi-implicit complex diffusion schemes for Optical Coherence Tomography despeckling

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Abstract. In this paper we illustrate the use of explicit and semi-implicit finite difference schemes for nonlinear complex diffusion in the context of medical imaging despeckling, namely for Optical coherence Tomography images. Differente boundary conditions will also be used. Performance metrics are shown to illustrate the feasibility of the numerical schemes and compare its results.

Keywords: Optical Coherence Tomography, denoising, despeckling, complex diffusion, finite differences

1 Introduction

Diffusion processes are commonly used for image processing in general, such as impaiting, denoising and stereo vision [3, 4, 6–8, 10–12]. Particular applications for image denoising and despeckling are of interested [3, 5, 7, 8] and have been used in the past decade. The these methods are usually based in the discretization of a nonlinear diffusion equation of the form

$$\frac{\partial u}{\partial t}(x,t) = \nabla \cdot (D(x,t,u)\nabla u(x,t)), \qquad \text{in } Q \times [0,T] \tag{1}$$

where the solution $u(x,t)$ represents different stages of the filtered image, x is the spatial coordinate defined in the square $Q = [1, N_1] \times [1, N_2]$ (where the inital image as dimensions $N_1 \times N_2$), t is the time coordinate defined in the interval $[0,T]$ where T is the diffusion time, and the diffusion coefficient D has to be properly defined in order to avoid diffusion across intensity edges in the image and therefore a blurring effect. Perona and Malik [7] proposed to use a diffusion coefficient based in the gradient of the image, in order to distinguish between edges and constant regions. However, in the initial stpes od the image where the noise level is high, the gradient is unstable. To overcome this problem, Gilboa [5] suggested to consider an appropriate complex filter of the form

$$D = \frac{e^{i\vartheta}}{1 + \left(\frac{\mathrm{Im}(u)}{\kappa\vartheta}\right)^2}. \tag{2}$$

where $\vartheta \approx 0$ and κ is a positive coefficient. It was proven that this filter is efficient, since

$$\lim_{\vartheta \to 0} \frac{\mathrm{Im}(u(.,t))}{t\theta} = G * \Delta I_0$$

where I_0 is the initial image, G is a gaussian and $*$ holds for the convolution operator. In [3], the method was adapted for the case o Optical coherence tomography (OCT) images. In [1] rigorous stability results were proven for finite difference schemes for (1) with complex diffusion coefficient. The convergence result was also achieved in [2].

Though implicit and semi-implicit finite differences methods are shown to be unconditionally stable, engineers tend to implement and use explicit schemes for the diffusion process. The explicit scheme is easier to implement, but the step in time is limited by the stability condition, leading to the need of a higher number of time steps required. On the other hand, implicit and semi-implicit schemes need to solve a linear system at every time step, though the number of time steps can be considerably lower.

In this work we will compare explicit and semi-implicit schemes for complex diffusion of OCt images in terms of image metrics. In this way, we aim at showing how these versions perform in terms of denoising metrics. Moreover, we will consider both Dirichlet and Neumann boundary conditions for the equation (1), in order to illustrate if it has an effect on the denoising of the image.

The paper is organized as follows. In section 2 we present the numerical schemes and the considered boundary conditions. In section 3 we present the main results, showing denoising metrics of each condidered approach. Finally, we draw some conclusions and future perspectives in section 4.

2 Methods

Let I_0 be the original (noisy) image of size $N_1 \times N_2$. Complex diffusion is usually based in the numerical solution of the partial differential equation (1). In order to have a well posed problem, equation (1) must be complemented by an initial condition of the form

$$u(x, 0) = I_0, \tag{3}$$

where I_0 is the original (noisy) image. Moreover, one needs boundary conditions, defined in the boundary Γ of the set Q. We will consider Dirichlet boundary conditions

$$u(x,t) = g(x,t), \quad x \in \varGamma,\ t \in [0,T], \tag{4}$$

which means that the boundaries of the image are kept fixed with some given g. In alternative, we will also consider Neumann boundary conditions

$$\frac{\partial u}{\partial \nu}(x,t) = 0, \quad x \in \varGamma,\ t \in [0,T], \tag{5}$$

which means the there is no normal intensity flux, where ν is the unit exterior normal vector.

2.1 Discretization

We consider an equally spaced mesh on Q. Let $h_1 = h_2 = 1$ be the mesh spacement in the first and second spatial coordinate direction. The mesh is therefore defined by the set of points

$$x_{j,k} = (j,k), \quad i = 0,1,2,\dots,N_1+1,\ k = 0,1,2,\dots,N_2+1.$$

Moreover, we consider the a time spacement $h_{t,m}$ which defines the set of points

$$t_{m+1} = t_m + h_{t,m}, \quad m = 0,1,\dots,N_t.$$

such that $t_{N_t} = T$. Thefore we define a mesh Q_h defined by the set of points

$$(x_{j,k}, t_m), \quad i = 0,1,2,\dots,N_1+1,\ k = 0,1,2,\dots,N_2+1,\ m = 0,1,\dots,N_t.$$

We consider the general finite difference scheme (see eq.(2.9) in [1]) for the differential equation (1) given by

$$\begin{aligned}
U_{j,k}^{m+1} = U_{j,k}^{m} &+ \frac{h_{t,m}}{2h_1^2}\Big[\left(D_{(j+1,k)}^{m,\theta,\mu} + D_{(j,k)}^{m,\theta,\mu}\right)U_{(j+1,k)}^{m+\theta} + \left(D_{(j,k)}^{m,\theta,\mu} + D_{(j-1,k)}^{m,\theta,\mu}\right)U_{(j-1,k)}^{m+\theta} \\
&\qquad - \left(D_{(j+1,k)}^{m,\theta,\mu} + 2D_{(j,k)}^{m,\theta,\mu} + D_{(j-1,k)}^{m,\theta,\mu}\right)U_{(j,k)}^{m+\theta}\Big] \\
&+ \frac{h_{t,m}}{2h_2^2}\Big[\left(D_{(j,k+1)}^{m,\theta,\mu} + D_{(j,k)}^{m,\theta,\mu}\right)U_{(j,k+1)}^{m+\theta} + \left(D_{(j,k)}^{m,\theta,\mu} + D_{(j,k-1)}^{m,\theta,\mu}\right)U_{(j,k-1)}^{m+\theta} \\
&\qquad - \left(D_{(j,k+1)}^{m,\theta,\mu} + 2D_{(j,k)}^{m,\theta,\mu} + D_{(j,k-1)}^{m,\theta,\mu}\right)U_{(j,k)}^{m+\theta}\Big]
\end{aligned} \tag{6}$$

for $j = 1,2,\dots,N_1$, $k = 1,2,\dots,N_2$, $m = 0,1,\dots,N_t-1$ where $U_{j,k}^m$ is the approximation to the solution $u(x_{j,k}, t_m)$,

$$D_{(j,k)}^{m,\theta,\mu} = D\left(x_{j,k}, t^{m+\theta}, U_{j,k}^{m+\mu}\right),$$

and $\mu \in \{0,1\}$, $t^{m+\theta} = \theta t^{m+1} + (1-\theta)t^m$, $\theta \in [0,1]$ and

$$V_{j,k}^{m+\theta} = \theta V_{j,k}^{m+1} + (1-\theta)V_{j,k}^{m}, \quad V = U, D,$$

Several choices of (θ, μ) give several different numerical schemes.

We will consider the explicit scheme given by $\theta = 0$ and $\mu = 0$, that is,

$$\begin{aligned}U_{j,k}^{m+1} = \Bigg[1 - \frac{h_{t,m}}{2h_1^2}\left(D_{(j+1,k)}^m + 2D_{(j,k)}^m + D_{(j-1,k)}^m\right) \\ - \frac{h_{t,m}}{2h_2^2}\left(D_{(j,k+1)}^m + 2D_{(j,k)}^m + D_{(j,k-1)}^m\right)\Bigg] U_{j,k}^m \\ + \frac{h_{t,m}}{2h_1^2}\left[\left(D_{(j+1,k)}^m + D_{(j,k)}^m\right)U_{(j+1,k)}^m + \left(D_{(j,k)}^m + D_{(j-1,k)}^m\right)U_{(j-1,k)}^m\right] \\ + \frac{h_{t,m}}{2h_2^2}\left[\left(D_{(j,k+1)}^m + D_{(j,k)}^m\right)U_{(j,k+1)}^m + \left(D_{(j,k)}^m + D_{(j,k-1)}^m\right)U_{(j,k-1)}^m\right]\end{aligned} \tag{7}$$

for $j = 1, 2, \ldots, N_1$, $k = 1, 2, \ldots, N_2$, $m = 0, 1, \ldots, N_t - 1$. This scheme is known to be stable (see [1]) for

$$h_{t,m} \leq \frac{\min(h_1, h_2)}{4 \max \frac{\left|D_{j,k}^m\right|^2}{\mathrm{Re}\left(D_{j,k}^m\right)}}.$$

We will also consider the semi-implicit scheme given by $\theta = 1$ and $\mu = 0$. This scheme is known to be unconditionally stable (see [2]), so one can choose a fixed step in time $h_t = T/N_t$, that is, one as the numerical scheme

$$\begin{aligned}\Bigg[1 + \frac{h_t}{2h_1^2}\left(D_{(j+1,k)}^m + 2D_{(j,k)}^m + D_{(j-1,k)}^m\right) \\ + \frac{h_t}{2h_2^2}\left(D_{(j,k+1)}^m + 2D_{(j,k)}^m + D_{(j,k-1)}^m\right)\Bigg] U_{j,k}^{m+1} \\ - \frac{h_t}{2h_1^2}\left[\left(D_{(j+1,k)}^m + D_{(j,k)}^m\right)U_{(j+1,k)}^{m+1} + \left(D_{(j,k)}^m + D_{(j-1,k)}^m\right)U_{(j-1,k)}^{m+1}\right] \\ - \frac{h_t}{2h_2^2}\left[\left(D_{(j,k+1)}^m + D_{(j,k)}^m\right)U_{(j,k+1)}^{m+1} + \left(D_{(j,k)}^m + D_{(j,k-1)}^m\right)U_{(j,k-1)}^{m+1}\right] \\ = U_{j,k}^m,\end{aligned} \tag{8}$$

for $j = 1, 2, \ldots, N_1$, $k = 1, 2, \ldots, N_2$, $m = 0, 1, \ldots, N_t - 1$. For this numerical schemme, a linear system needs to be solved at each iteration. As an iterative method is advised (for computation efficiency) we consider the preconditioned conjugate gradient algorithm of MatLab.

One also needs to consider the initial condition

$$U_{j,k}^0 = I_0(j,k), \quad j = 1, 2, \ldots, N_1,\ k = 1, 2, \ldots, N_2. \tag{9}$$

We also consider the Dirichlet boundary condition

$$\begin{aligned}U_{j,0}^m = I_0(j,1),\ U_{j,N_2+1}^m = I_0(j,N_1), \quad j = 1, 2, \ldots, N_1, \\ U_{0,k}^m = I_0(1,k),\ U_{N_1+1,k}^m = I_0(N_1,k), \quad k = 1, 2, \ldots, N_2,\end{aligned} \tag{10}$$

for $m = 0, 1, \ldots, N_t$ or the Neumann condition

$$\begin{aligned} U^m_{j,0} = U^m_{j,2},\, U^m_{j,N_2+1} = U^m_{j,N_2-1}, &\quad j = 1, 2, \ldots, N_1, \\ U^m_{0,k} = U^m_{2,k},\, U^m_{N_1+1,k} = U^m_{N_1-1,k}, &\quad k = 1, 2, \ldots, N_2, \end{aligned} \tag{11}$$

In this work we will compare the performances of the following numerical schemes:

1. Explicit method with Dirichlet boundary conditions defined by the system of equations (7), (9), (10);
2. Explicit method with Neumann boundary conditions defined by the system of equations (7), (9), (11);
3. Semi-implicit method with Dirichlet boundary conditions defined by the system of equations (8), (9), (10);
4. Semi-implicit method with Neumann boundary conditions defined by the system of equations (8), (9), (11);

2.2 Diffusion coefficient

We consider the complex diffusion coefficient (2) proposed by Gilboa [5]. While Gilboa considered κ constant, an improved version for OCT filtering was proposed in [3] as

$$\kappa = \kappa_{MIN} + (\kappa_{MAX} - \kappa_{MIN})\frac{g - \min(g)}{\max(g) - \min(g)}$$

where $\kappa_{MIN}, \kappa_{MAX}$ are given real constants with $\kappa_{MIN} < \kappa_{MAX}$ and

$$g = G_{N,\sigma} * \mathrm{Re}(u),$$

where $G_{N,\sigma}$ is a gaussian of size $N \times N$ and standard deviation σ and $*$ holds for the convolution operator.

2.3 Performance metrics

To test the performance of the proposed filtering methods, we consider both synthetic and real OCT images.

In the synthetic images [9], one creates a noisefree synthetic OCT image and adds OCT-like speckle noise. In this way it makes sense to use metrics like the mean square error (MSE)

$$\mathrm{MSE} = \frac{\displaystyle\sum_{j=1}^{N_1}\sum_{k=1}^{N_2}\left(I(i,j) - U^{N_t}_{j,k}\right)^2}{N_1 N_2}$$

where I is noisefree image, and the mean structure similarity index [**?**]

$$MSSIM = \frac{1}{N}\sum_{j,k} SSIM(v_I(j,k), v_U(j,k)) \tag{12}$$

with

$$SSIM(v_1, v_2) = \frac{(2\mu_1\mu_2 + C_1)(2\sigma_{12} + C_2)}{(\mu_1^2 + \mu_2^2 + C_1)(\sigma_1^2 + \sigma_2^2 + C_2)} \tag{13}$$

where $v_I(j,k)$ and $v_U(j,k)$ represent local windows of the images I and U^{N_t}, respectively, in the neighborhood of the coordinate (j,k), $C_1 = (0.01L)^2$, $C_2 = (0.03L)^2$ and L is the maximum value allowed for the data (*e.g.*, 255 for 8-bit data). The values μ_j, σ_j and σ_{12} are given by

$$\begin{aligned}
\mu_j &= \sum_k w(k) v_j(k) \\
\sigma_j &= \sqrt{\sum_k w(k) \left(v_j(k) - \mu_j\right)^2} \\
\sigma_{12} &= \sum_k w(k) \left(v_1(k) - \mu_1\right)\left(v_2(k) - \mu_2\right)
\end{aligned} \tag{14}$$

with $j = 1, 2$ and w a trivariate Gaussian weight function (of integral equal to one and standard deviation 1.5) [**?**].

For both sinthetic or real OCT filtered images we also computed the effective number of looks (ENL), the signal to noise ratio (SNR) and the contrast to noise ratio (CNR) given by

$$\text{ENL} = \frac{\mu_H^2}{\sigma_H^2}, \quad \text{SNR} = \frac{\mu_H}{\sigma_H}, \quad \text{CNR} = 10\log\left(\frac{\mu_R - \mu_H}{\sqrt{\sigma_R^2 + \sigma_H^2}}\right),$$

where H is an homogeneous (background) region and R is a feature region in the image.

3 Results

We present the results in table 1. There are no significant differences in the filtering performance between the explicit and semi-implicit schemes, nor between the Neumann and Dirichlet boundary condition. There are significant differences for OCT filtering between κ constant or κ adaptive, has previously shown for the explicit Dirichlet case [3].

4 Conclusion

The performance in terms of filtering seems to be independent of the numerical scheme (explicit or semi-implicit) or the bounday condition (non-homogeneous Dirichelet or homogeneous Neumann). In this way, one can choose the best scheme in terms of computation eficiency. At this moment, this comparison has not yet been done, since the linear solver for the semi-implicit scheme can still be optimized.

Table 1. Metrics for the considered numerical schemes, using κ constant and adaptive.

T	scheme	κ	Bound. Cond.	CNR	ENL	MSE	MSSIM	SNR	CNR	ENL	SNR
0,1	explicit	constant	Dirichlet	5,7855	14,6426	48,7945	0,6579	1,7566			
	explicit	constant	Neumann	5,7865	14,6629	48,7411	0,6579	1,7568			
	explicit	adaptive	Dirichlet	5,7855	14,6426	48,7945	0,6579	1,7566			
	explicit	adaptive	Neumann	5,7865	14,6629	48,7411	0,6579	1,7568			
	semi-imp.	constant	Dirichlet								
	semi-imp.	constant	Neumann	5,3104	9,0524	78,9557	0,5144	1,6982			
	semi-imp.	adaptive	Dirichlet								
	semi-imp.	adaptive	Neumann	5,3178	9,1249	78,3746	0,5163	1,6993			
0,5	explicit	constant	Dirichlet	6,1188	22,638	32,7085	0,7484	1,7965			
	explicit	constant	Neumann	6,1156	22,4428	32,8474	0,7484	1,7963			
	explicit	adaptive	Dirichlet	5,876	13,1825	51,341	0,6098	1,7583			
	explicit	adaptive	Neumann	5,8725	13,0835	51,4071	0,6109	1,7582			
	semi-imp.	constant	Dirichlet								
	semi-imp.	constant	Neumann	6,0419	20,1384	35,4006	0,7397	1,786			
	semi-imp.	adaptive	Dirichlet								
	semi-imp.	adaptive	Neumann	6,0635	21,0712	33,9831	0,75	1,7893			
1	explicit	constant	Dirichlet								
	explicit	constant	Neumann								
	explicit	adaptive	Dirichlet								
	explicit	adaptive	Neumann								
	semi-imp.	constant	Dirichlet								
	semi-imp.	constant	Neumann								
	semi-imp.	adaptive	Dirichlet								
	semi-imp.	adaptive	Neumann								
5	explicit	constant	Dirichlet								
	explicit	constant	Neumann								
	explicit	adaptive	Dirichlet								
	explicit	adaptive	Neumann								
	semi-imp.	constant	Dirichlet								
	semi-imp.	constant	Neumann								
	semi-imp.	adaptive	Dirichlet								
	semi-imp.	adaptive	Neumann								

We have left the implicit method defined by $\theta = 1$ and $\mu = 1$ outside the scope of this comparison, since it needs a nonlinear solver at each step. The choice of the linearization process for the solver influences the result and will be object of future research.

Acknowledgment. P. Rodrigues, P. Guimarães, R. Bernardes and P. Serranho acknowledge the support by FCT under the research projects PTDC/SAU-BEB/103151/2008 and PTDC/SAU-ENB/111139/2009, and by the COMPETE programs FCOMP-01-0124-FEDER-010930 and FCOMP-01-0124-FEDER-015712. A. Araújo and S. Barbeiroackowledge the partial support by CMUC and FCT through European program COMPETE/FEDER and project UTAustin/MAT/0066/2008.

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